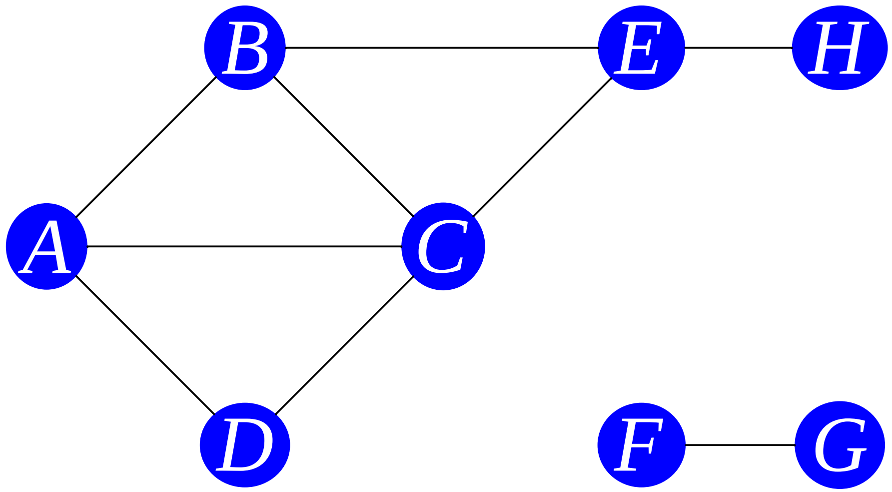
Graph theory

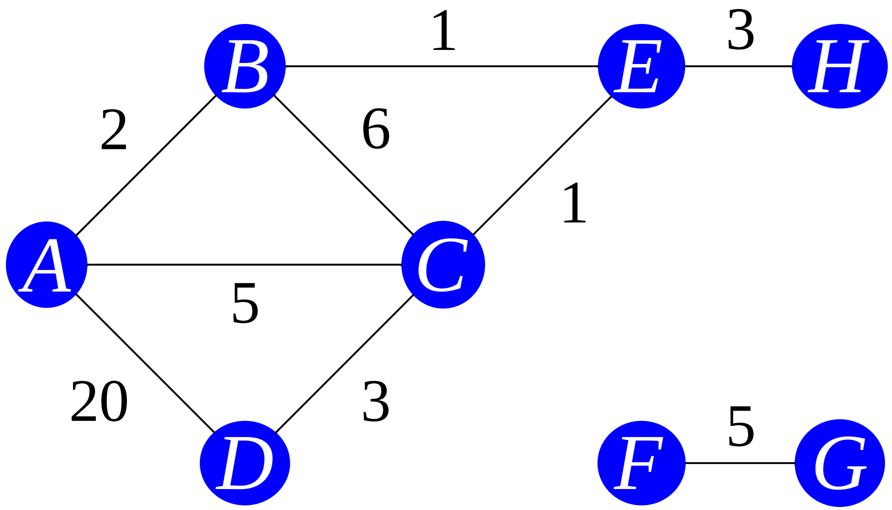
Nodes and edges

Bruce Merry

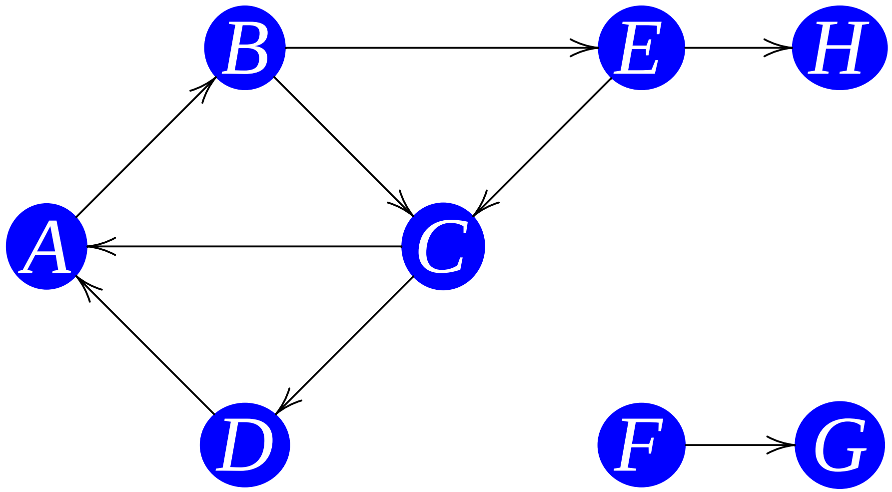
Simple graph



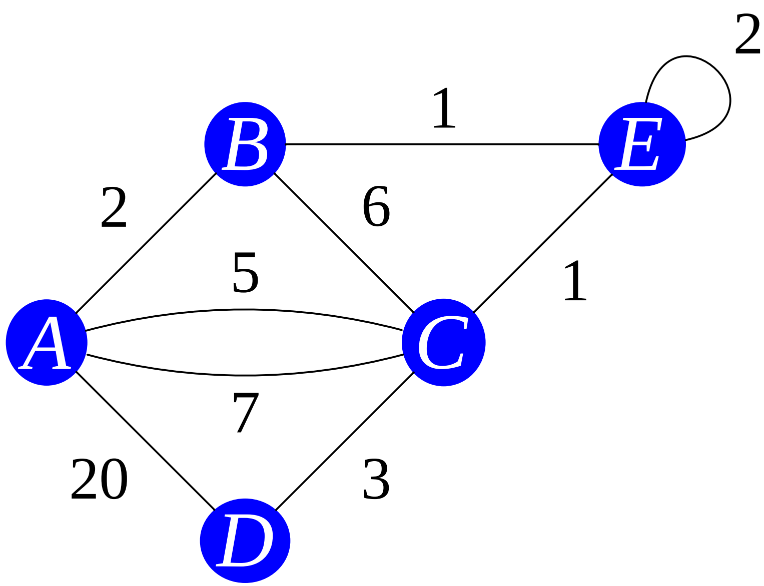
Weighted graph



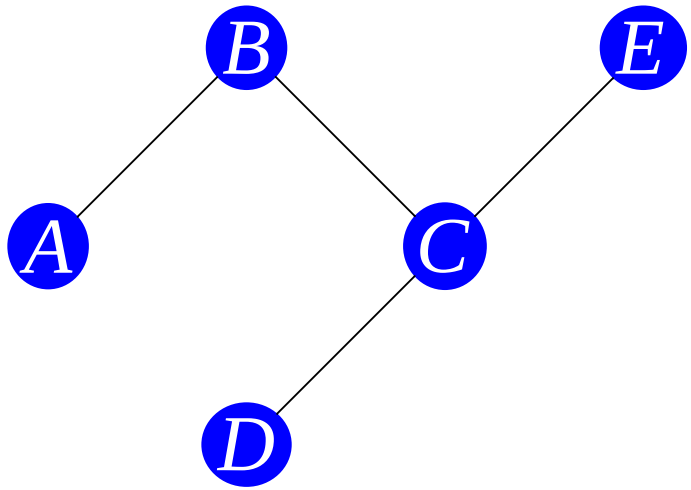
Directed graph



Multigraph

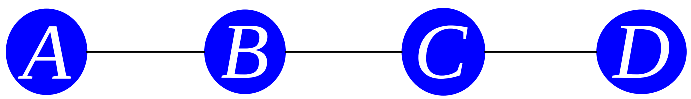


Tree



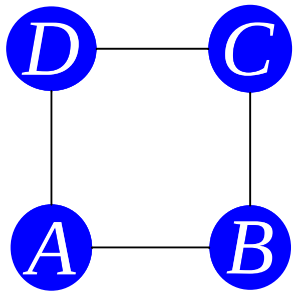
Other terminology

- path



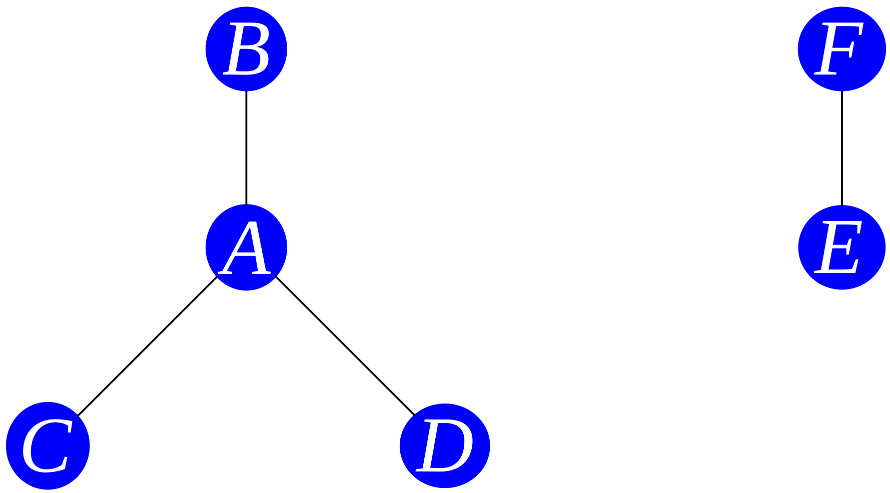
Other terminology

- path
- cycle



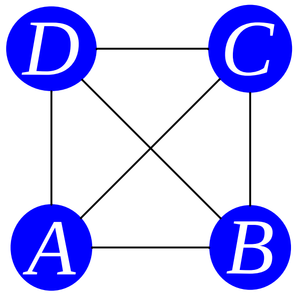
Other terminology

- path
- cycle
- connected



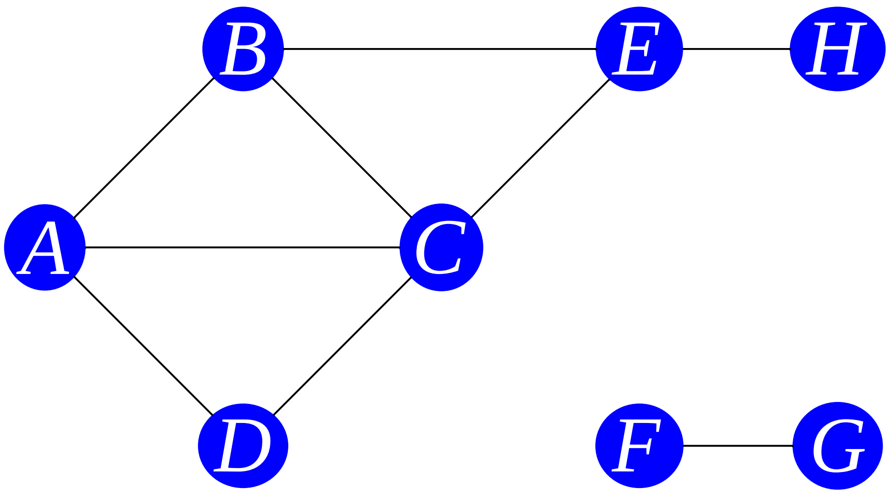
Other terminology

- path
- cycle
- connected
- complete



Other terminology

- path
- cycle
- connected
- complete
- degree

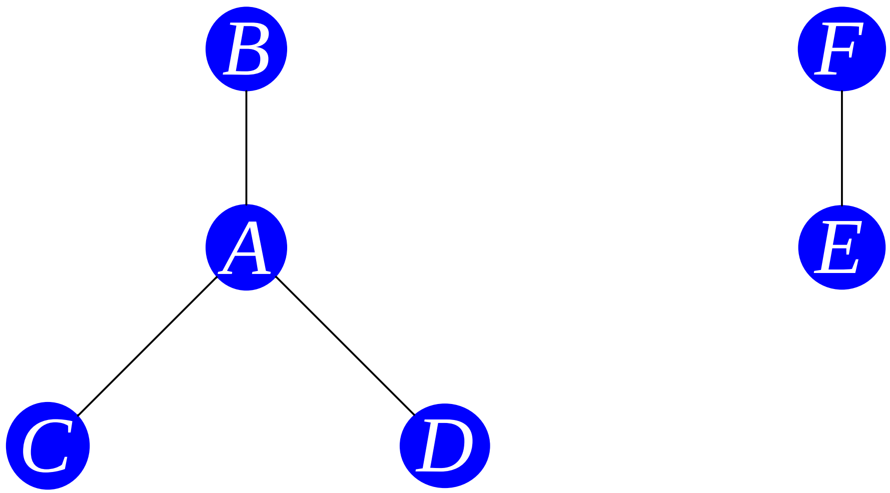


Other terminology

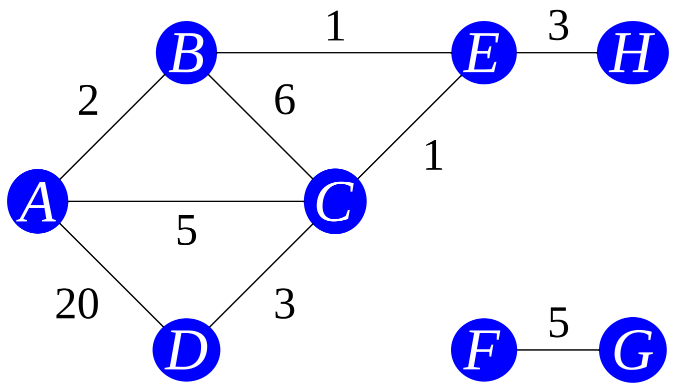
- path
- cycle
- connected
- complete
- degree
- dense and sparse

Other terminology

- path
- cycle
- connected
- complete
- degree
- dense and sparse
- forest

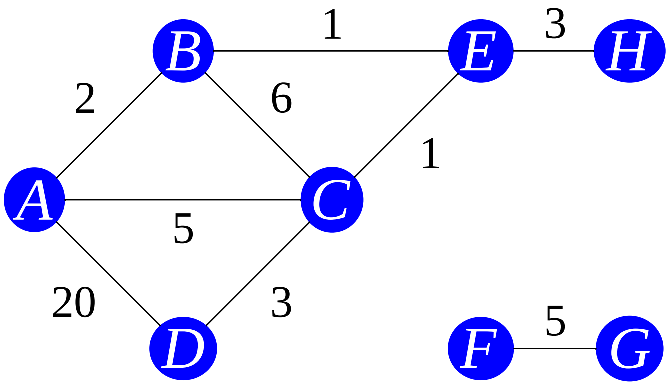


Representation



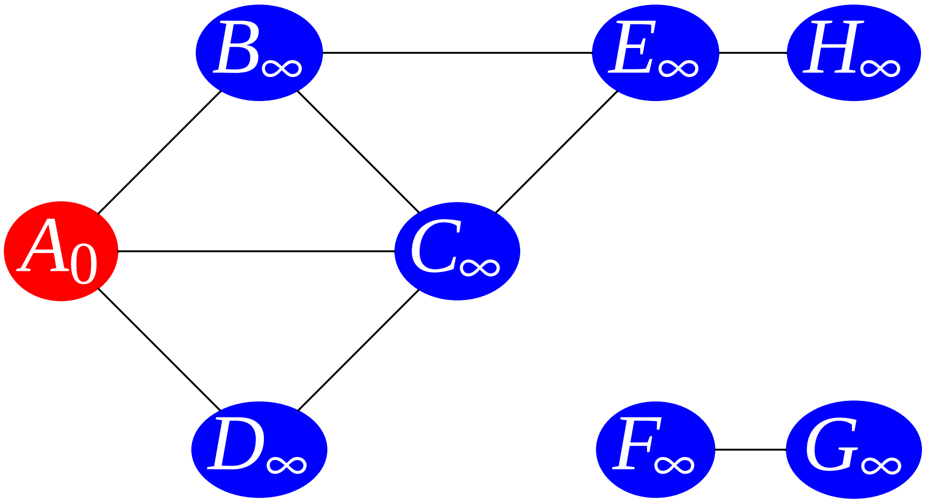
	A	B	C	D	E	F	G	H
A		2	5	20				
B	2		6		1			
C	5	6		3	1			
D	20		3					
E		1	1					3
F							5	
G						5		
H					3			

Representation



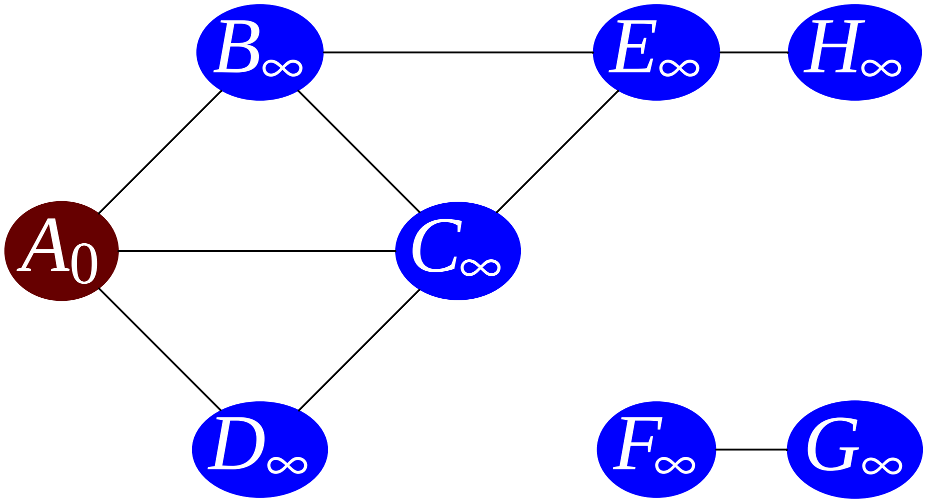
A	(B, 2) (C, 5) (D, 20)
B	(A, 2) (C, 6) (E, 1)
C	(B, 6) (A, 5) (D, 3) (E, 1)
D	(C, 3) (A, 20)
E	(B, 1) (C, 1) (H, 3)
F	(G, 5)
G	(F, 5)
H	(E, 3)

Breadth first search

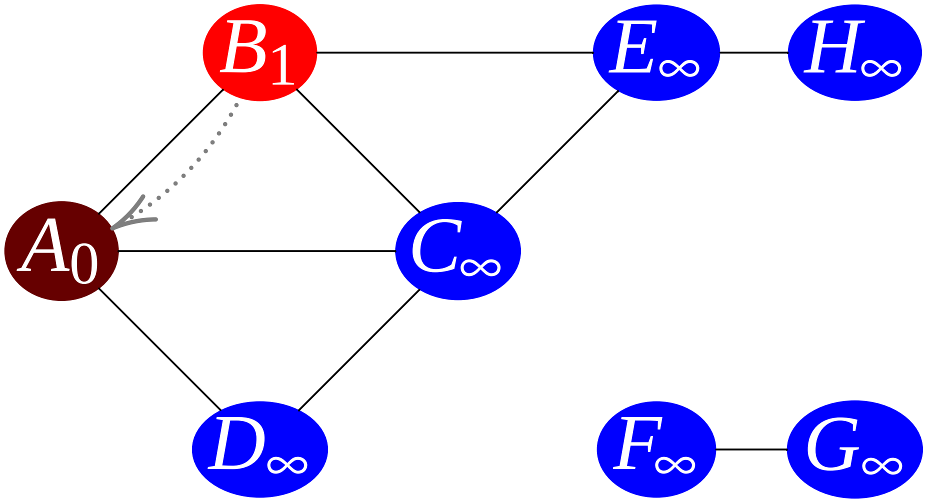


(A, 0)

Breadth first search

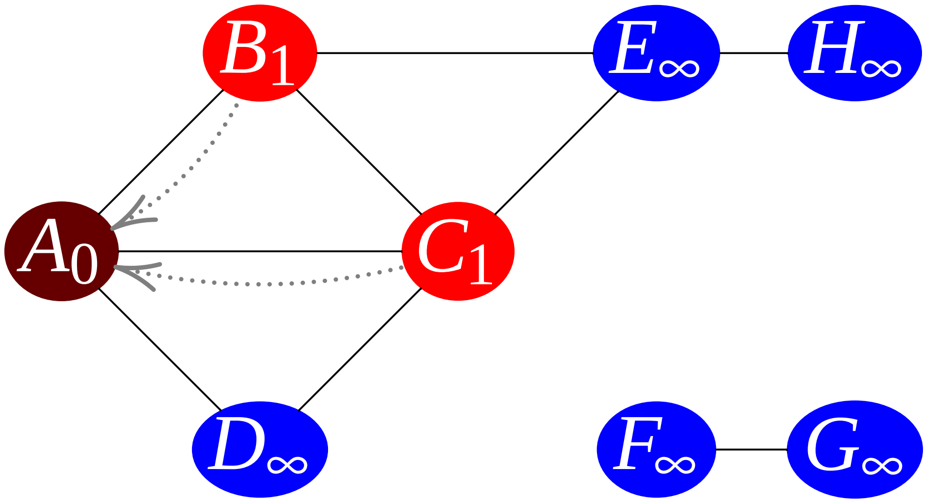


Breadth first search



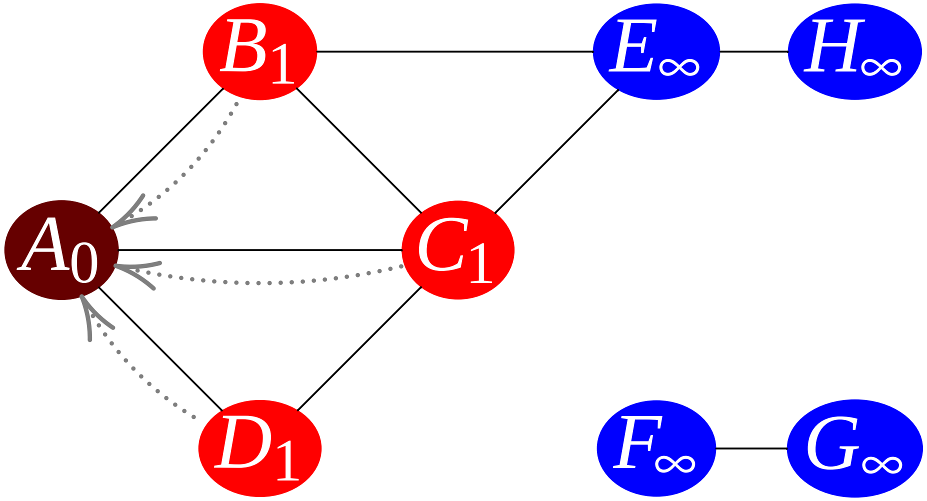
(B, 1)

Breadth first search



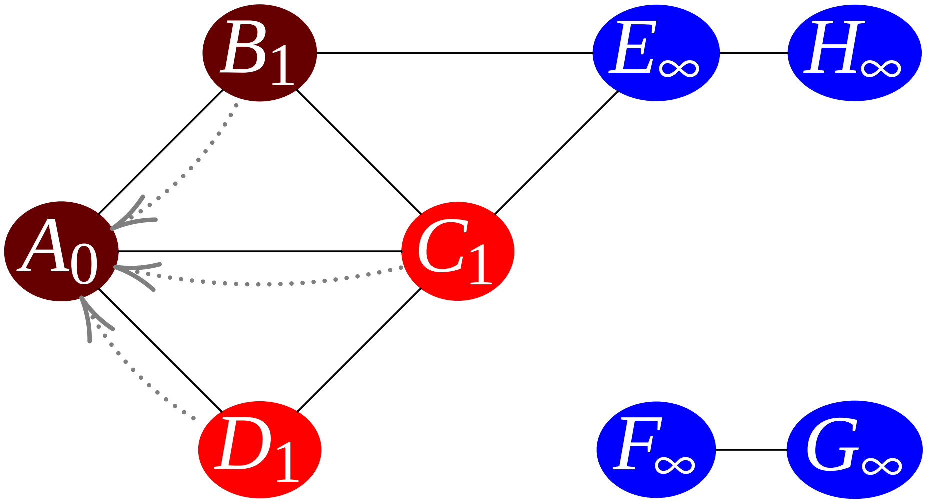
(B, 1)	(C, 1)

Breadth first search



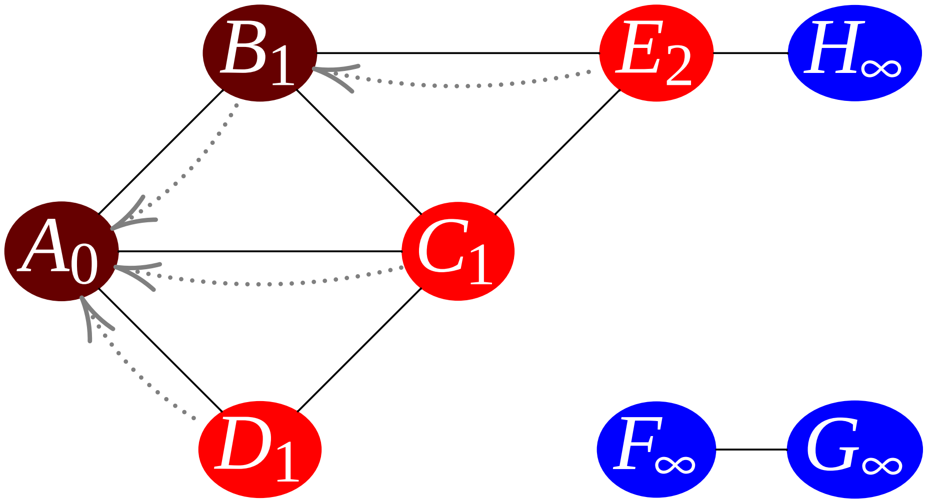
(B, 1)	(C, 1)	(D, 1)

Breadth first search



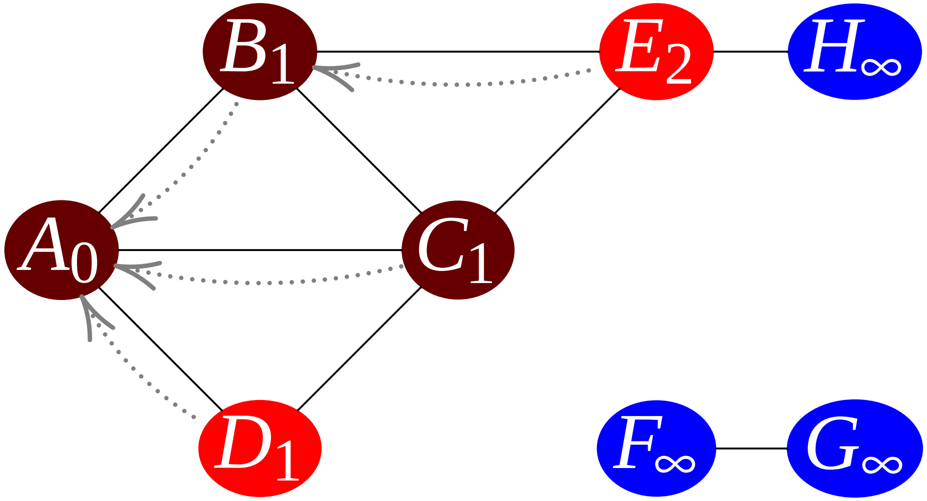
(C, 1)	(D, 1)

Breadth first search



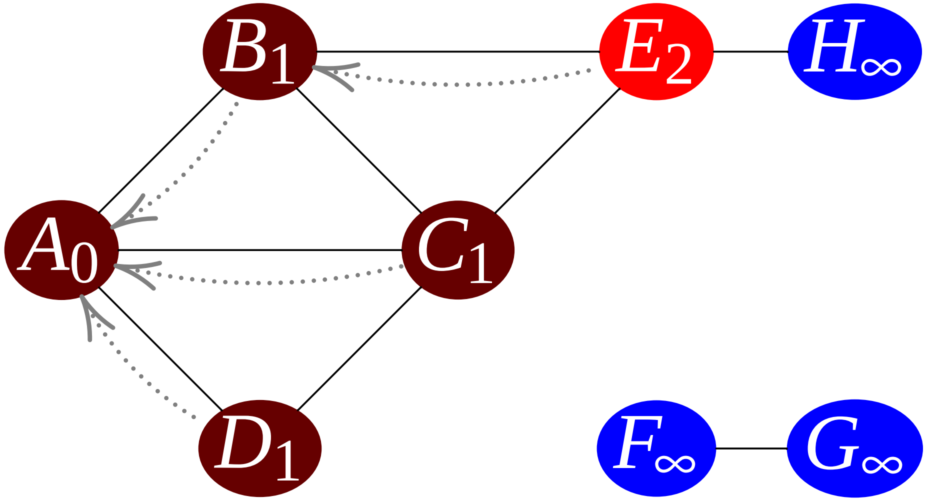
(C, 1)	(D, 1)	(E, 2)

Breadth first search



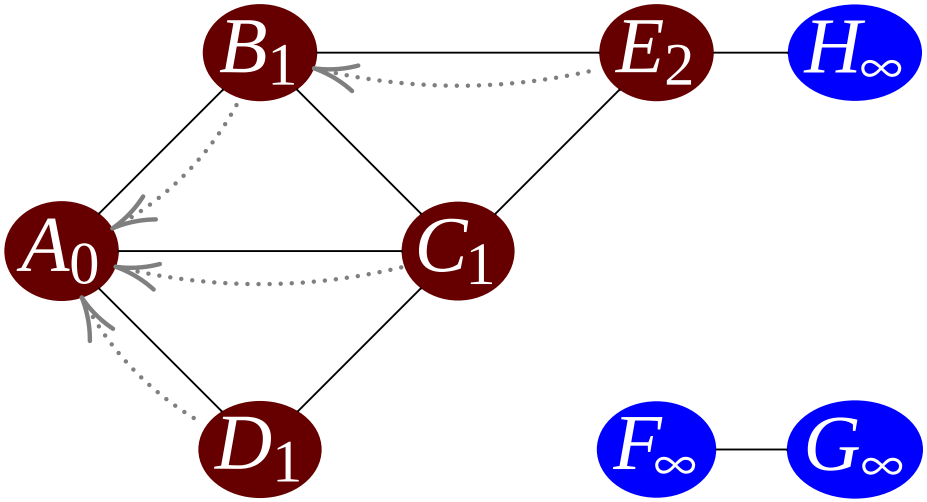
(D, 1)	(E, 2)

Breadth first search

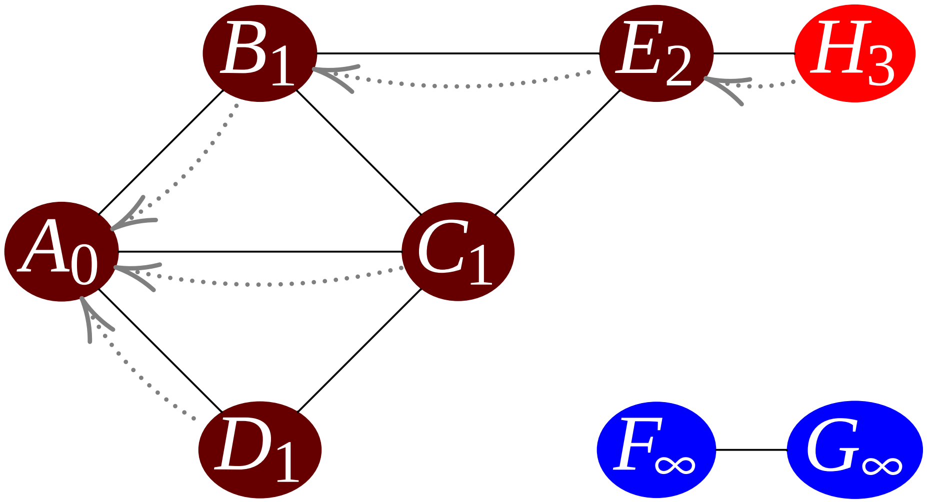


(E, 2)

Breadth first search

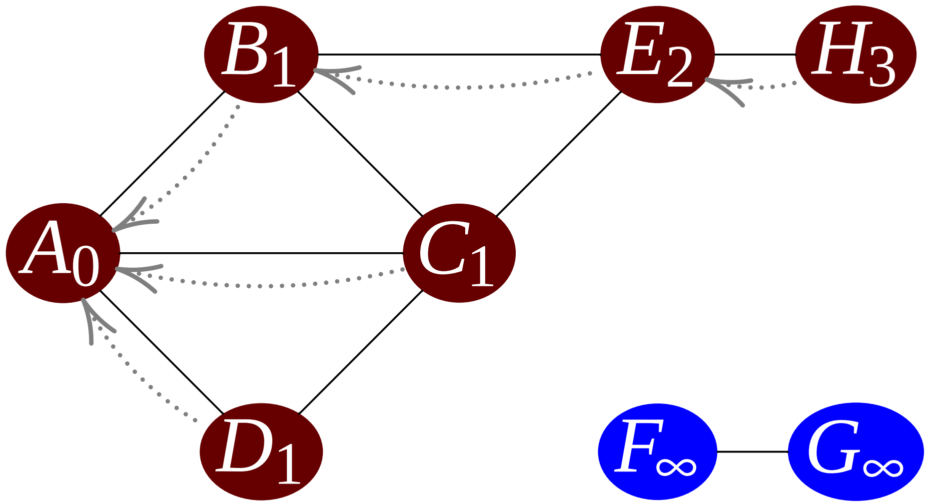


Breadth first search

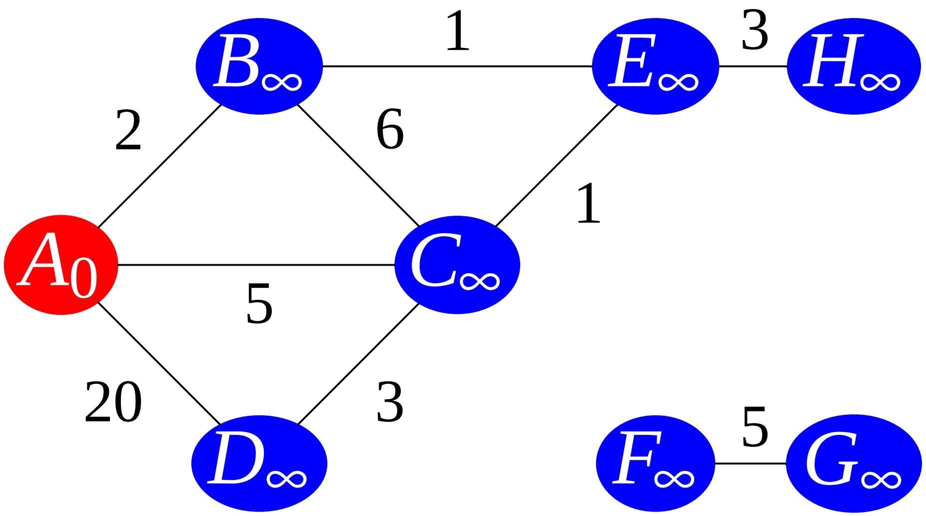


(H, 3)

Breadth first search

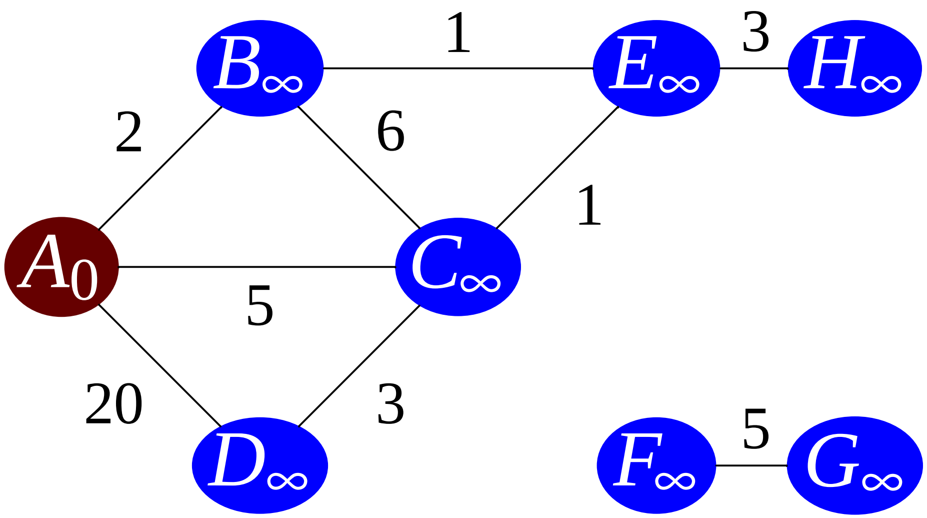


Dijkstra's algorithm

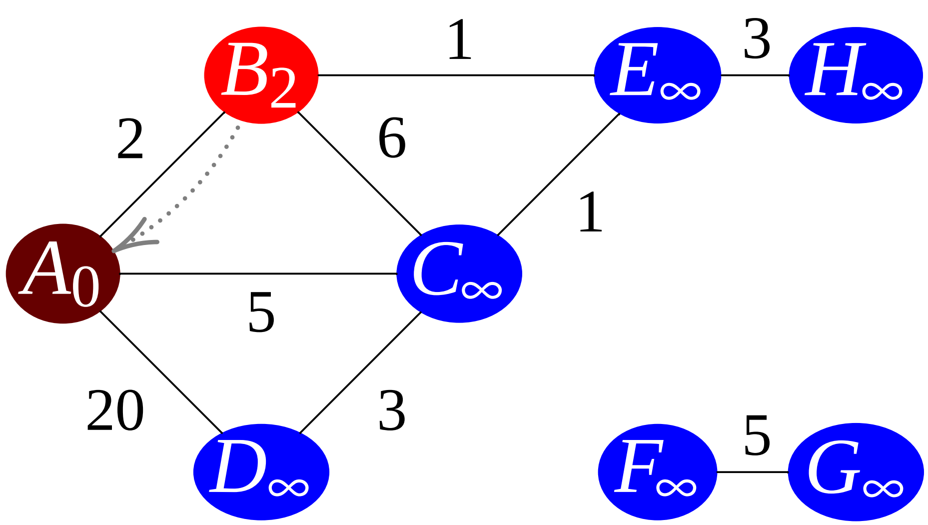


(A, 0)

Dijkstra's algorithm

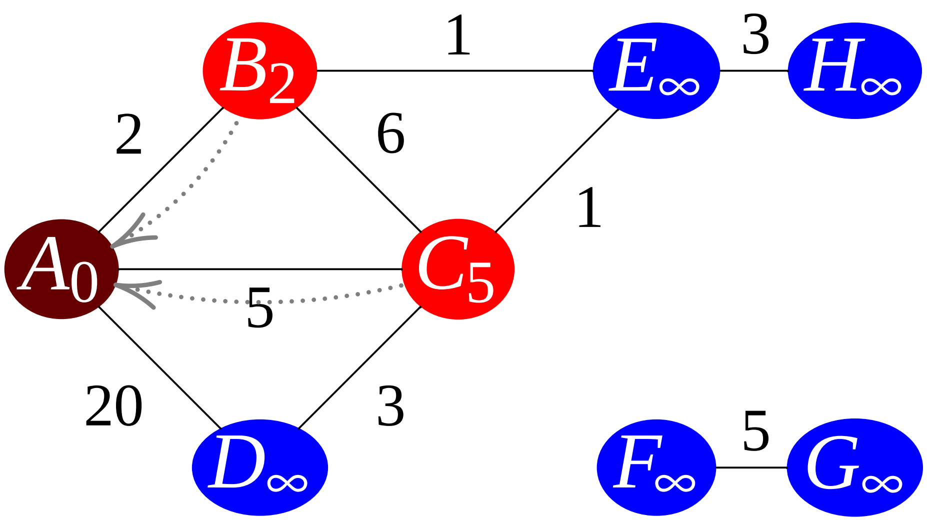


Dijkstra's algorithm



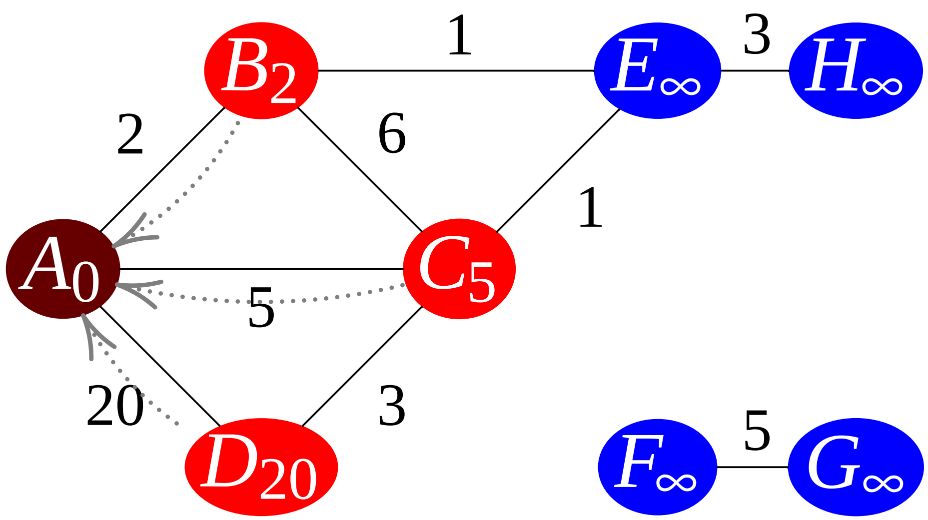
(B, 2)

Dijkstra's algorithm



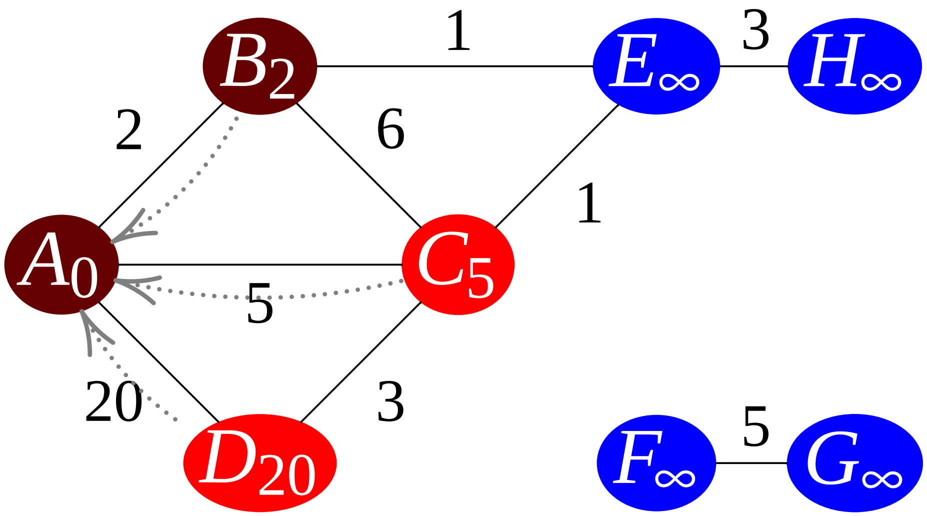
(B, 2)	(C, 5)

Dijkstra's algorithm



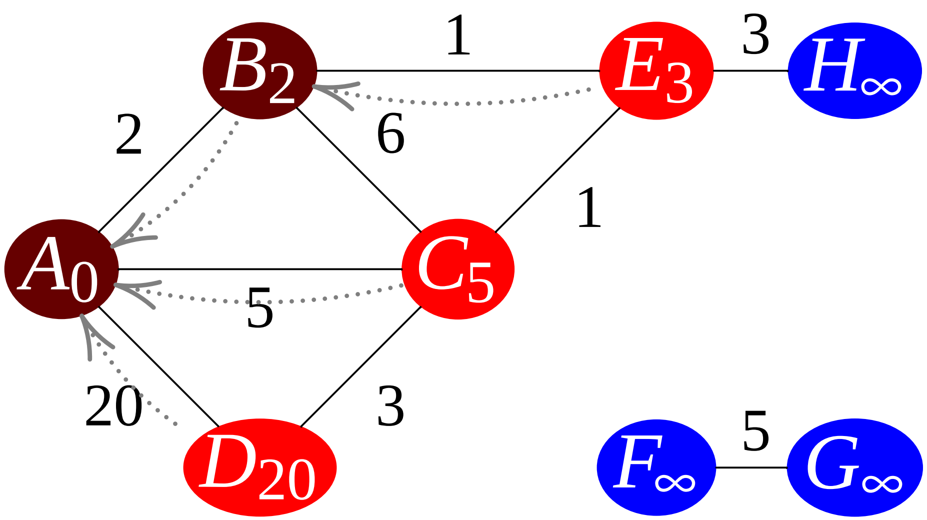
(B, 2)	(C, 5)	(D, 20)

Dijkstra's algorithm



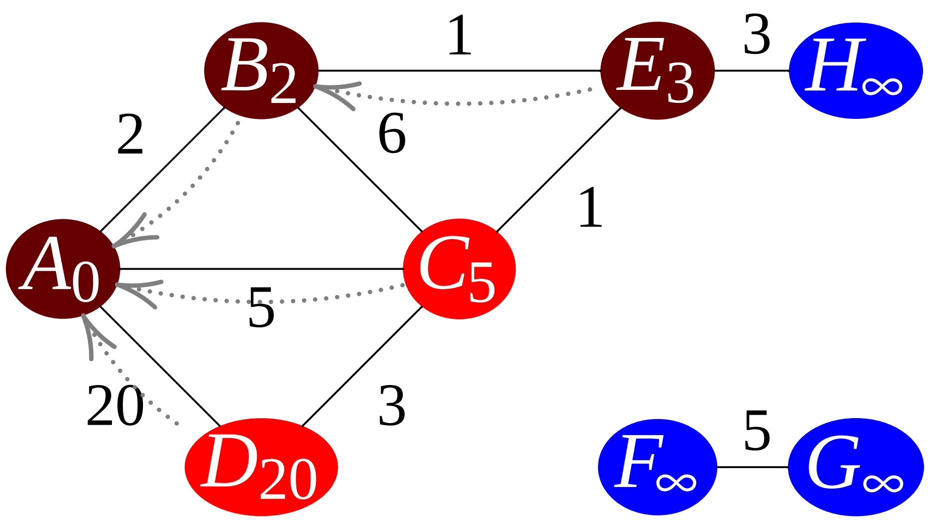
(C, 5)	(D, 20)

Dijkstra's algorithm



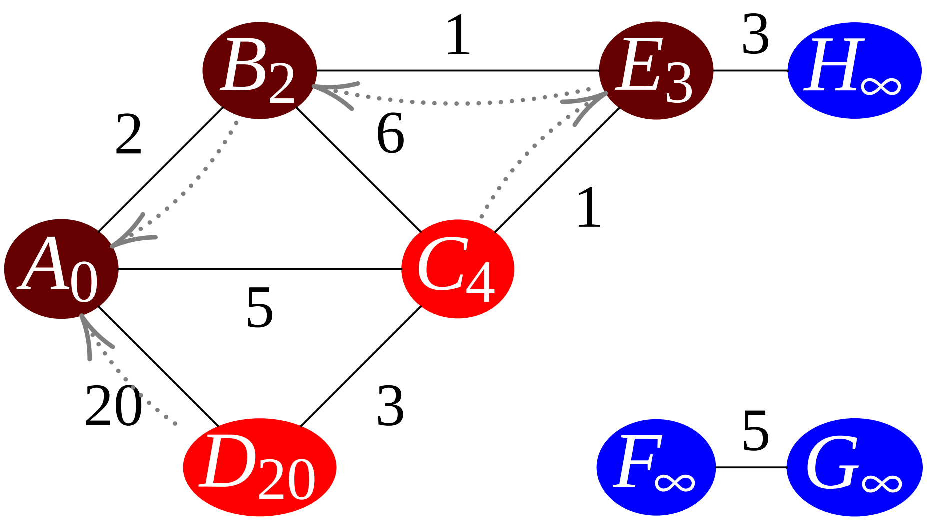
(E, 3)	(C, 5)	(D, 20)

Dijkstra's algorithm



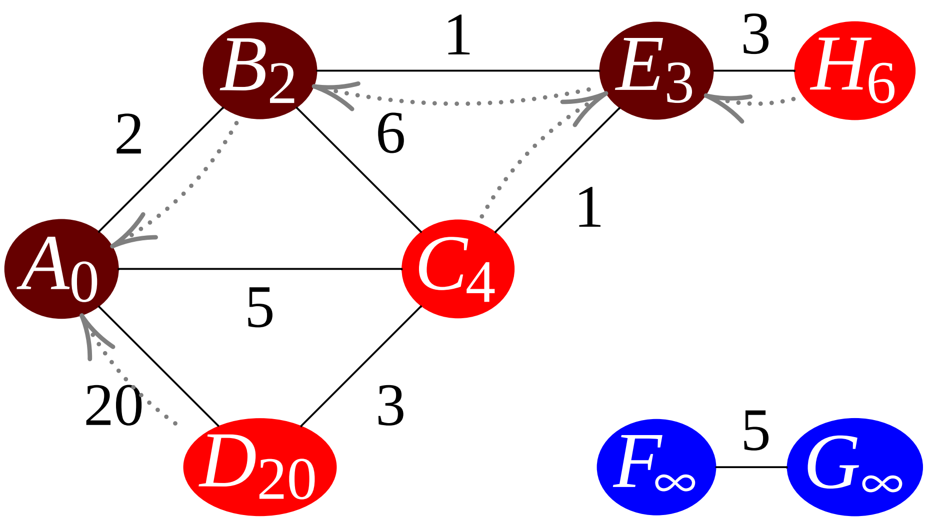
(C, 5)	(D, 20)

Dijkstra's algorithm



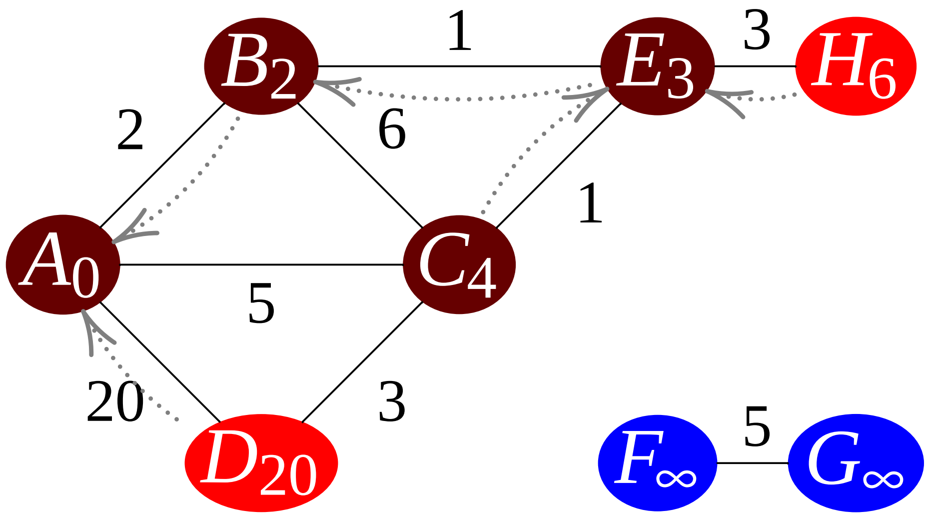
(C, 4)	(C, 5)	(D, 20)

Dijkstra's algorithm



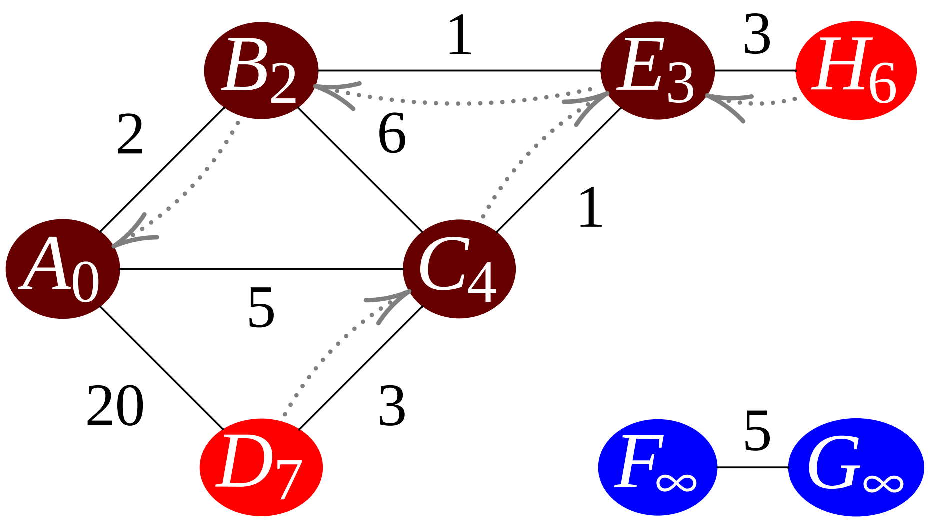
(C, 4)	(C, 5)	(H, 6)	(D, 20)

Dijkstra's algorithm



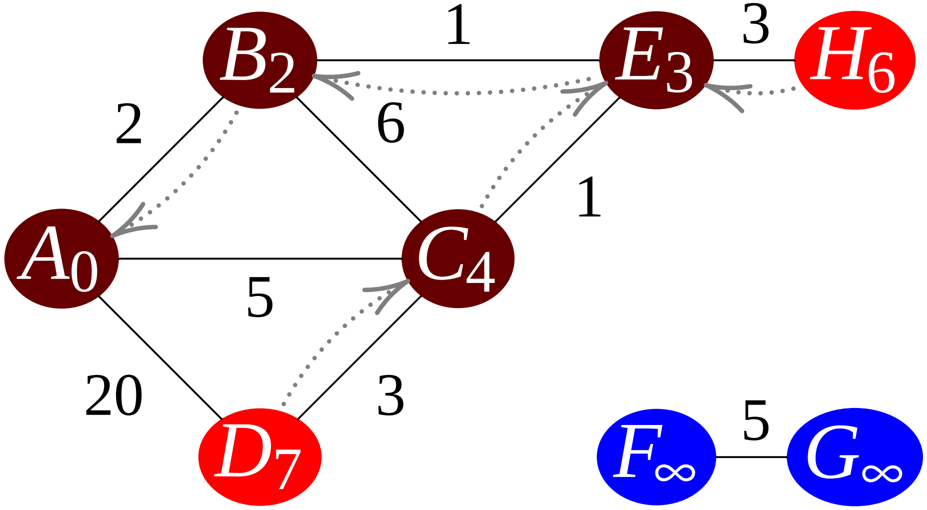
(C, 5)	(H, 6)	(D, 20)

Dijkstra's algorithm



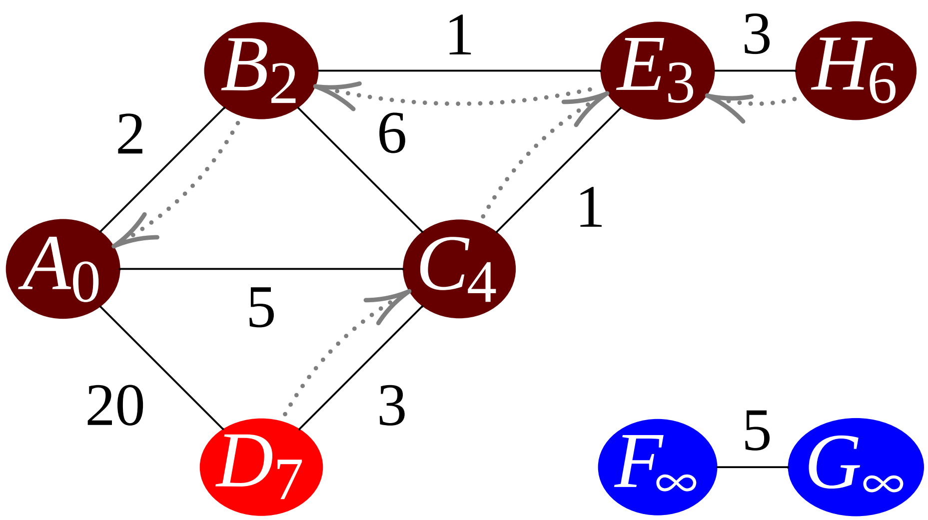
(C, 5)	(H, 6)	(D, 7)	(D, 20)

Dijkstra's algorithm



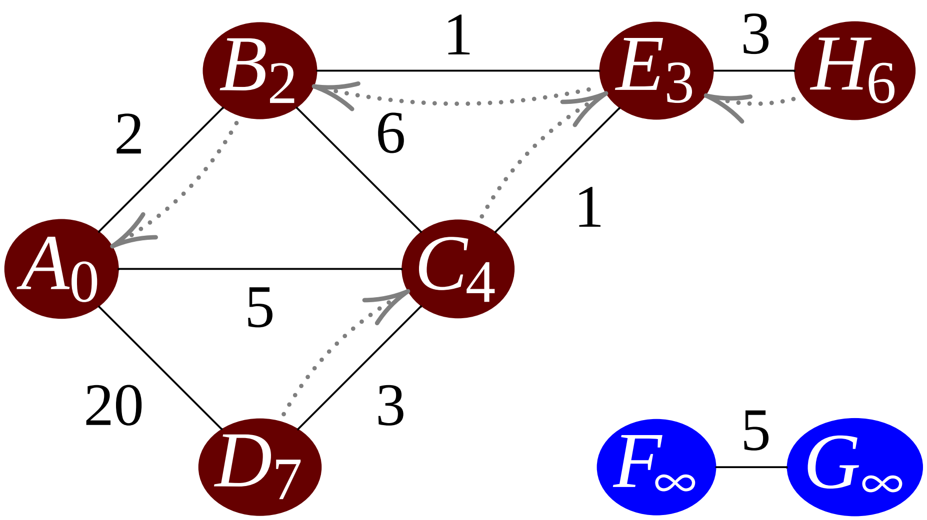
(H, 6)	(D, 7)	(D, 20)

Dijkstra's algorithm



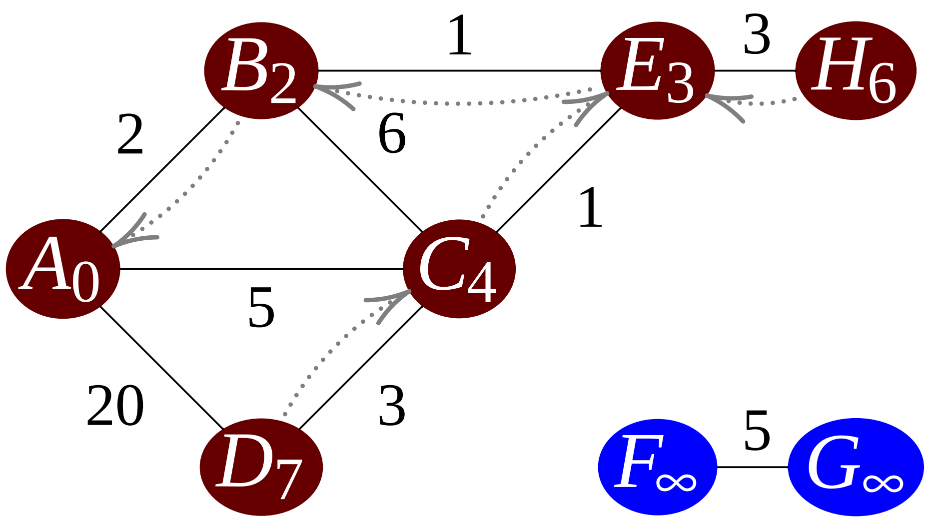
(D, 7)	(D, 20)

Dijkstra's algorithm



(D, 20)

Dijkstra's algorithm



Dijkstra's algorithm: efficiency

Unsorted list $\mathrm{O}(V^2)$ — easy and good for dense graphs

Heap $\mathrm{O}(E \cdot \log V)$ — trickier but good for sparse graphs

Sorted list $\mathrm{O}(VE)$ — good for nothing

Minimum spanning trees

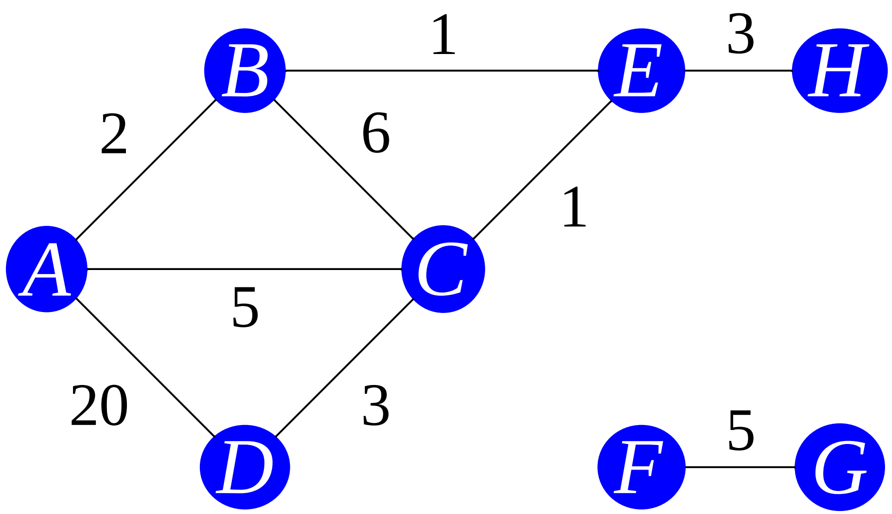
A *spanning tree* is a subset of the edges of a graph, which

- form a tree;
- touch every vertex of the original graph.

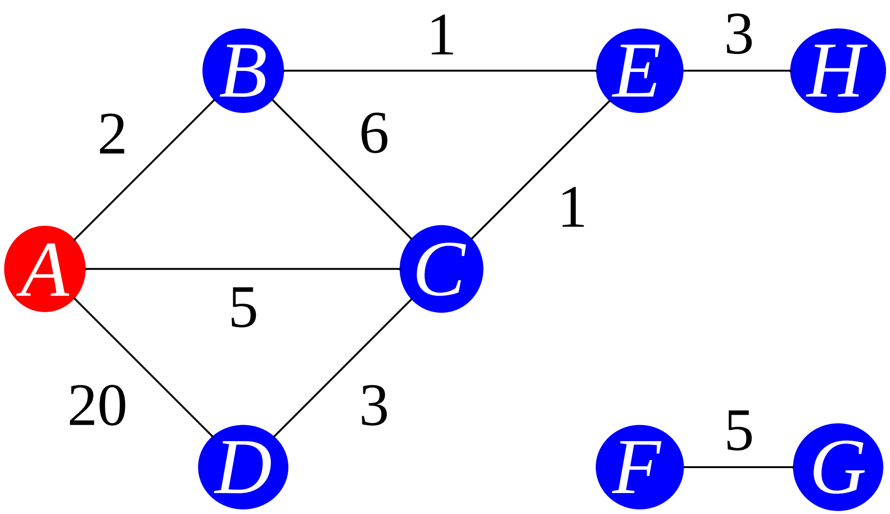
The *minimum spanning tree* has least total weight.

MST algorithms

Key observation: in any partition of the vertices, a shortest edge between the parts must connect them.

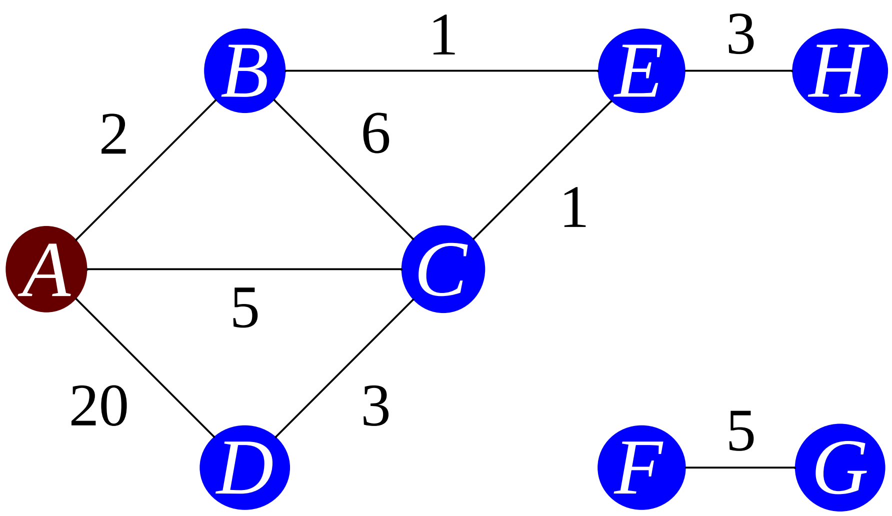


Prim's algorithm

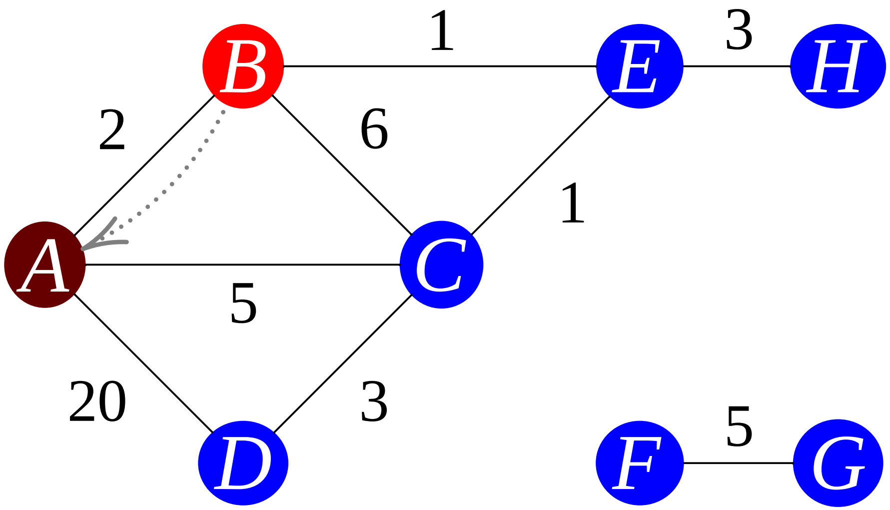


A

Prim's algorithm

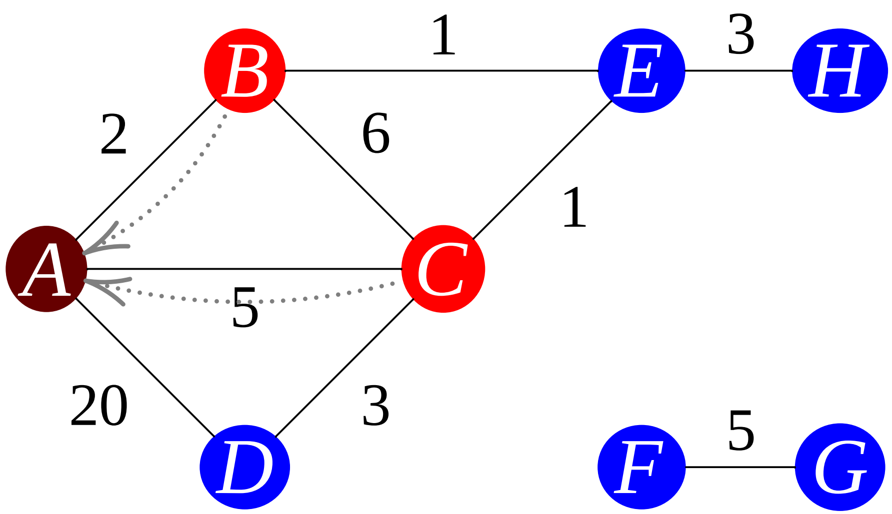


Prim's algorithm



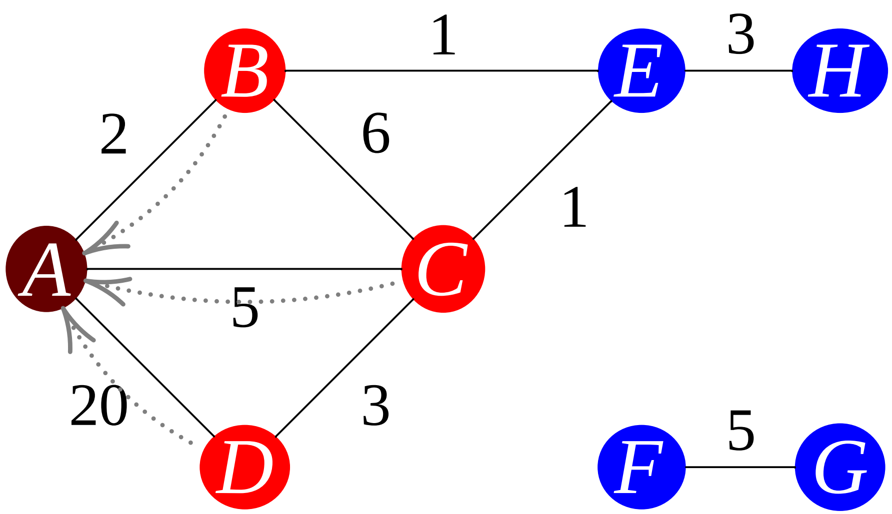
B

Prim's algorithm



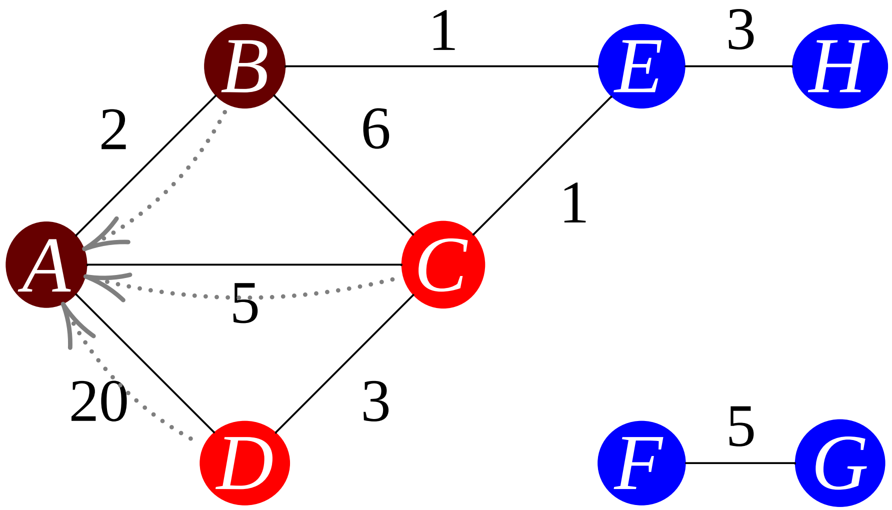
B	C

Prim's algorithm



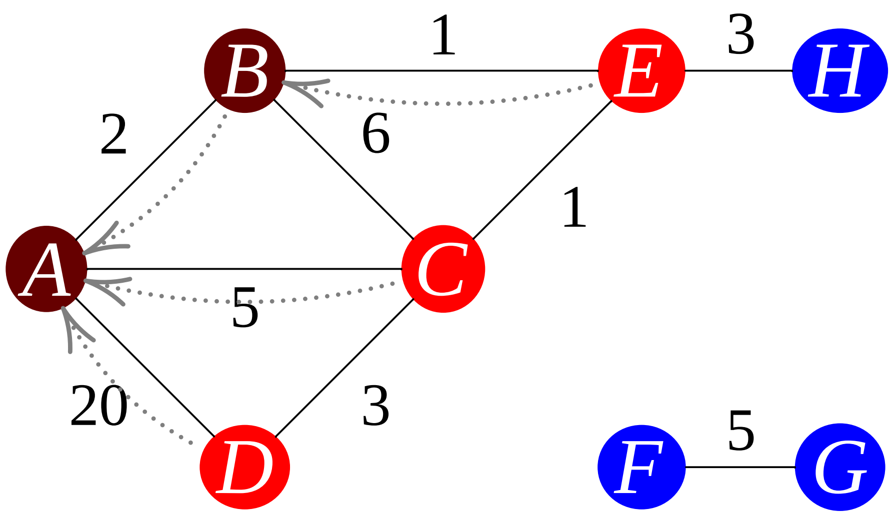
B	C	D

Prim's algorithm



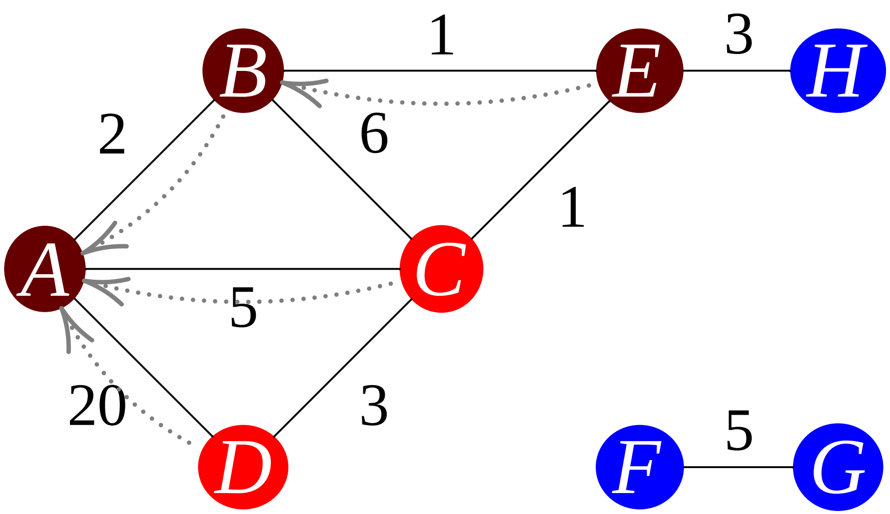
C	D

Prim's algorithm



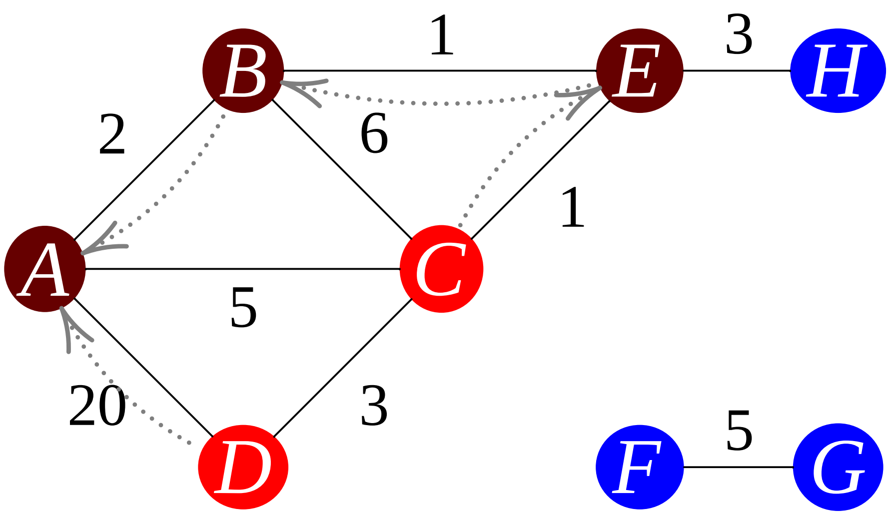
E	C	D

Prim's algorithm



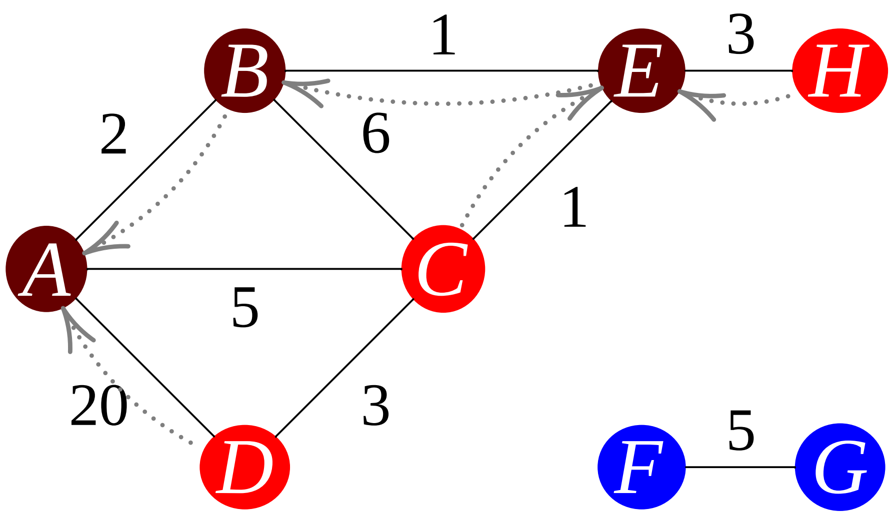
C	D

Prim's algorithm



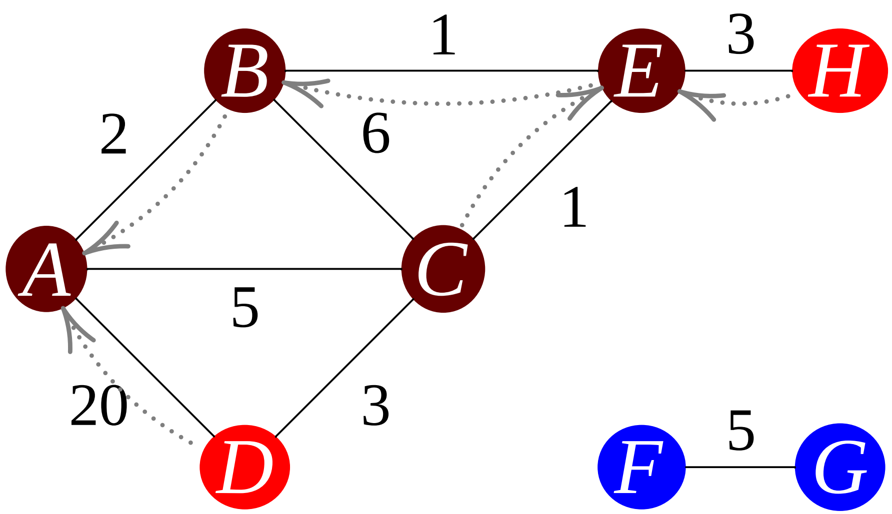
C	C	D

Prim's algorithm



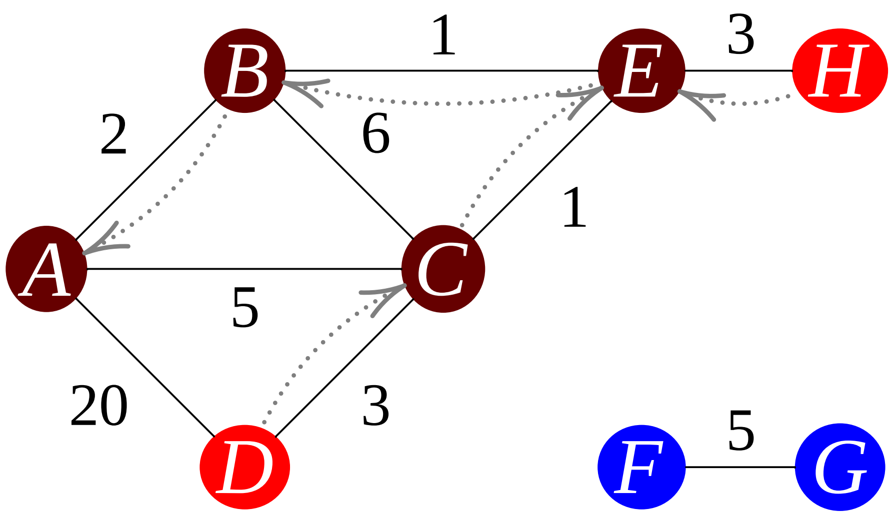
C	H	C	D

Prim's algorithm



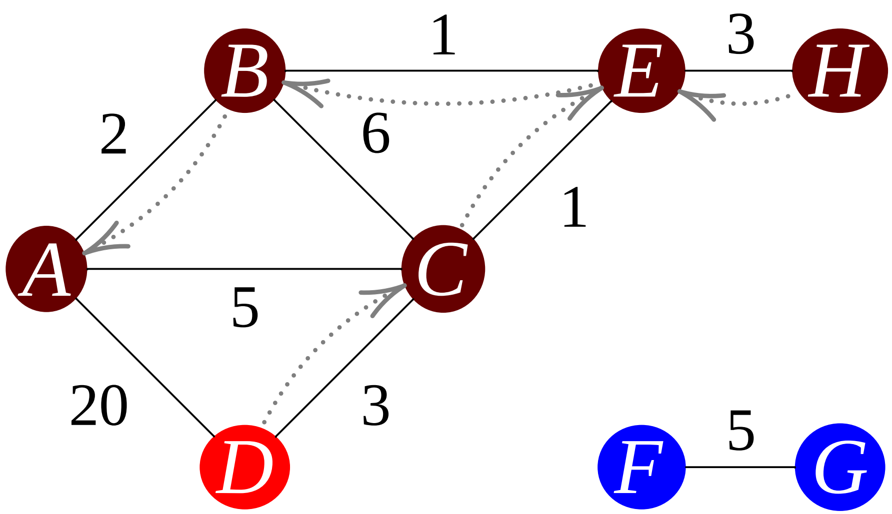
H	C	D

Prim's algorithm



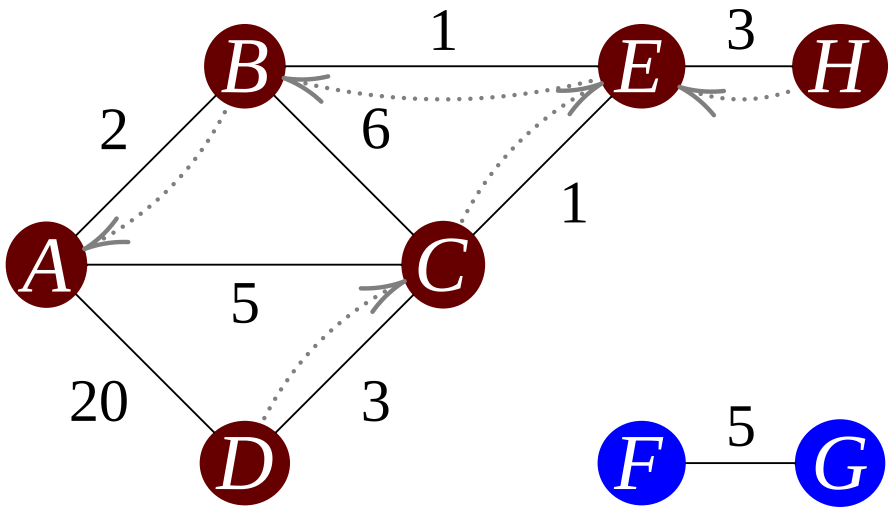
H	D	C	D

Prim's algorithm



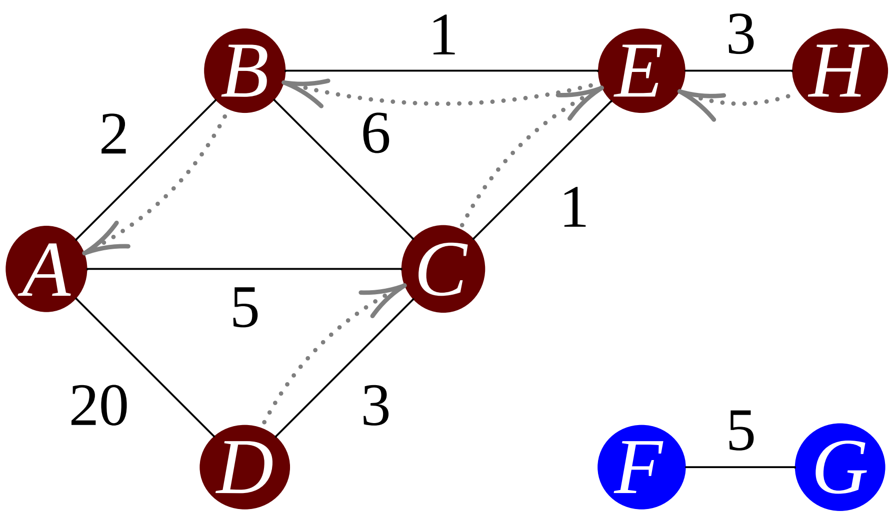
D	C	D

Prim's algorithm



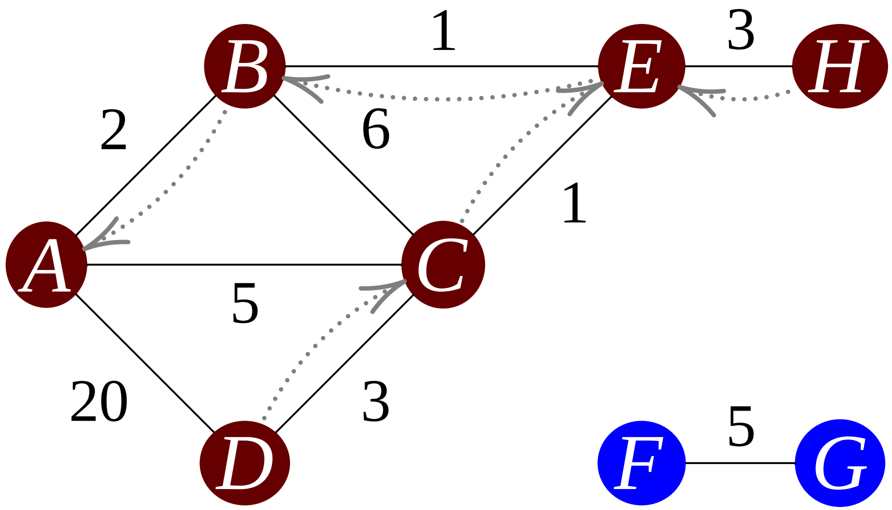
C	D

Prim's algorithm



D

Prim's algorithm



Floyd's algorithm

Let $x[y]z$ be the length of the shortest path from x to z, going only via $1, 2, \ldots, y$, or ∞ if no such edge exists. Then

- $x[0]z$ is the length of the edge from x to z
- $x[N]z$ is the shortest length from x to z
- $x[y]y = x[y-1]y$, $y[y]z = y[y-1]z$
- $x[y]z = \min\{x[y-1]z, x[y]y + y[y]z\}$

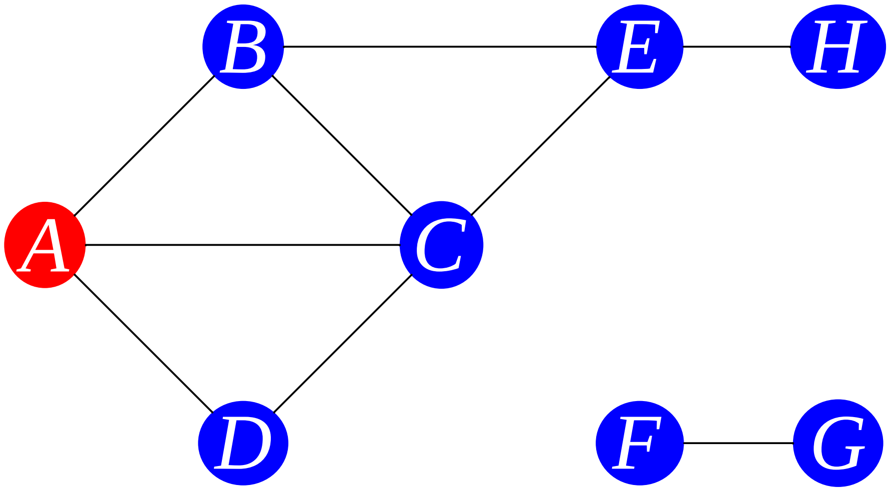
Start with table of $x[0]z$, then convert it to $x[1]z$, then to $x[2]z$ etc.

Floyd's algorithm

for $y = 1$ to N **do**
  for $x = 1$ to N **do**
    if matrix$[x][y] \neq \infty$ **then**
      for $z = 1$ to N **do**
        if matrix$[x][y] +$ matrix$[y][z] <$ matrix$[x][z]$
        then
          matrix$[x][z] \leftarrow$ matrix$[x][y] +$ matrix$[y][z]$

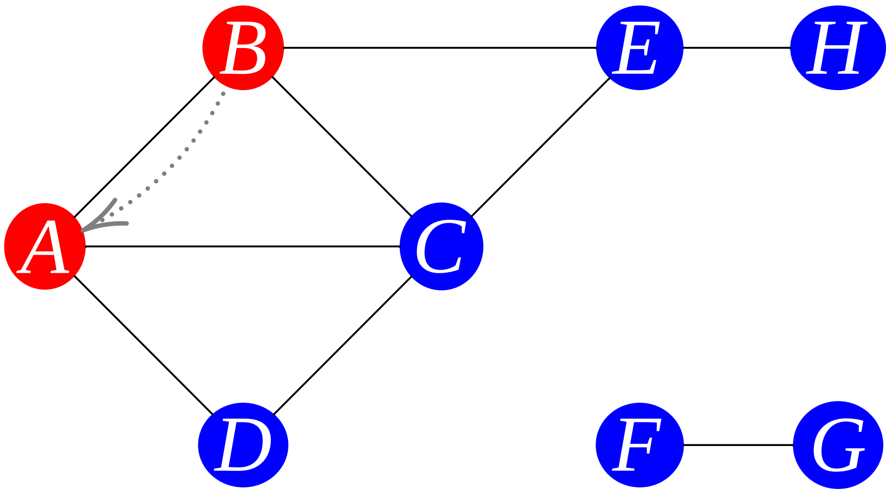
Efficiency: $O(V^3)$.

Depth first search



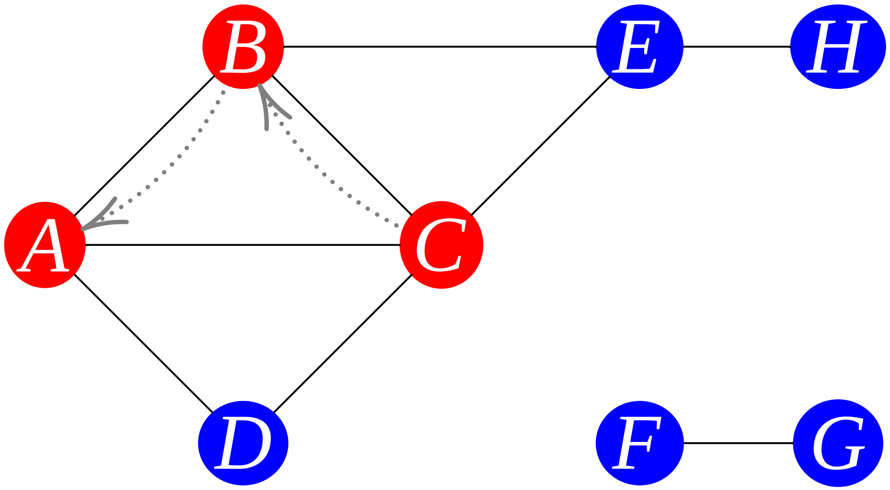
A

Depth first search



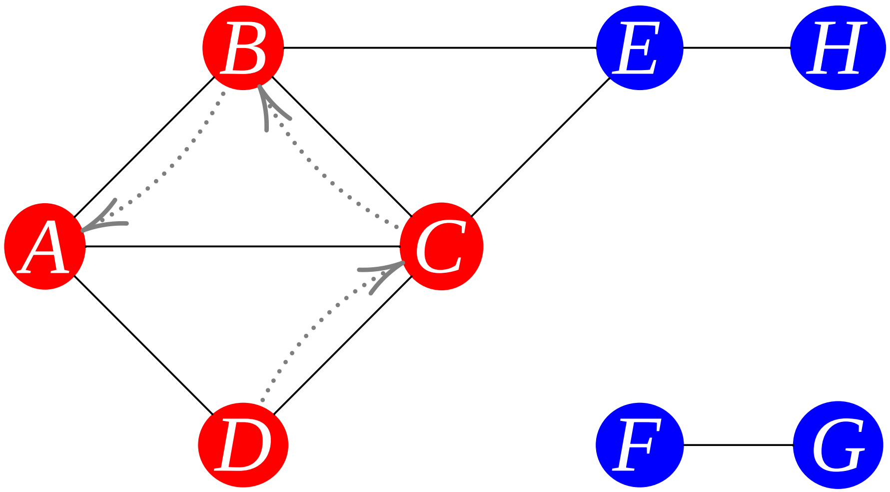
A	B

Depth first search



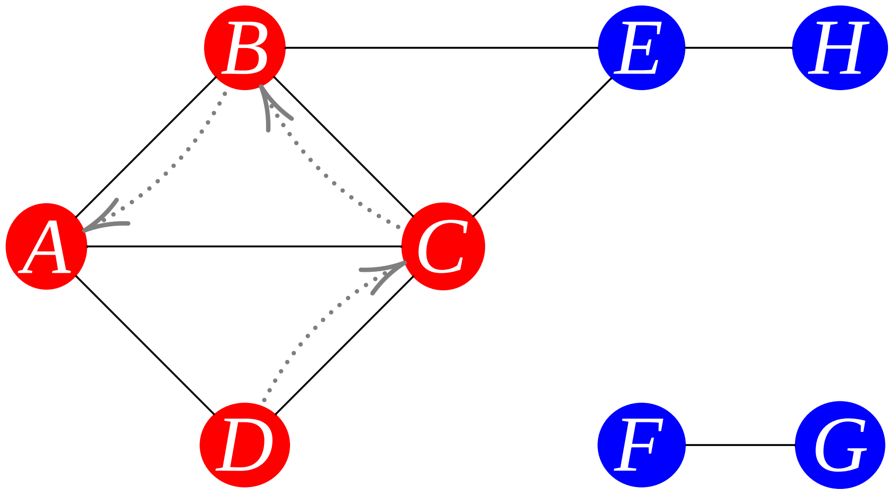
A	B	C

Depth first search



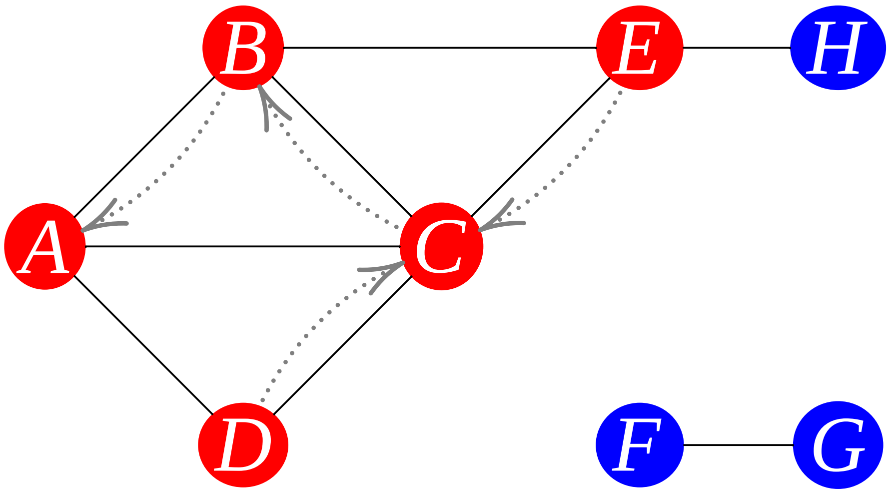
A	B	C	D

Depth first search



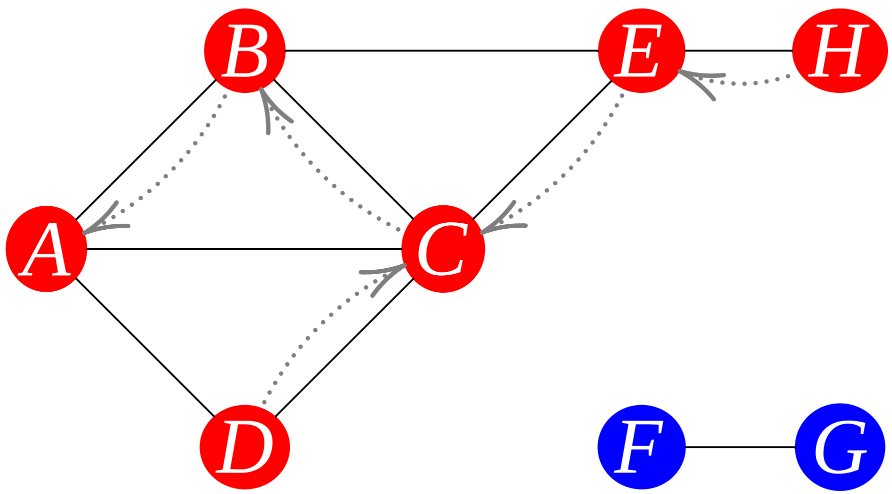
A	B	C

Depth first search



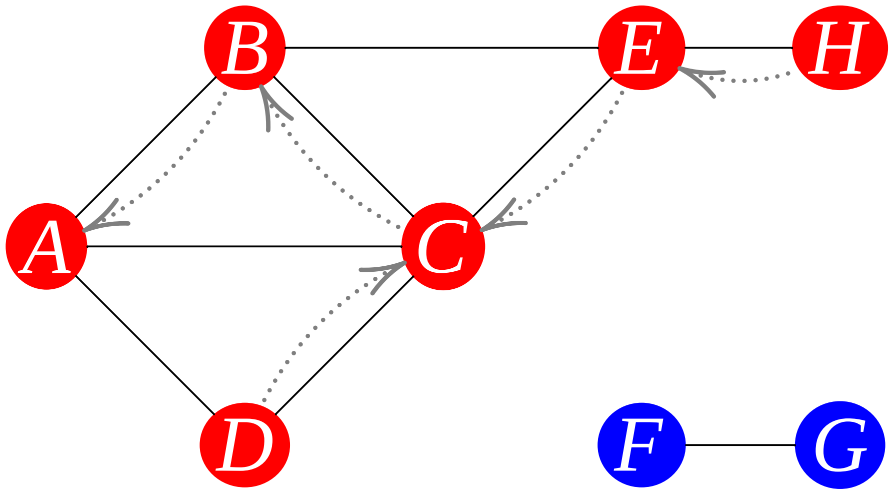
A	B	C	E

Depth first search



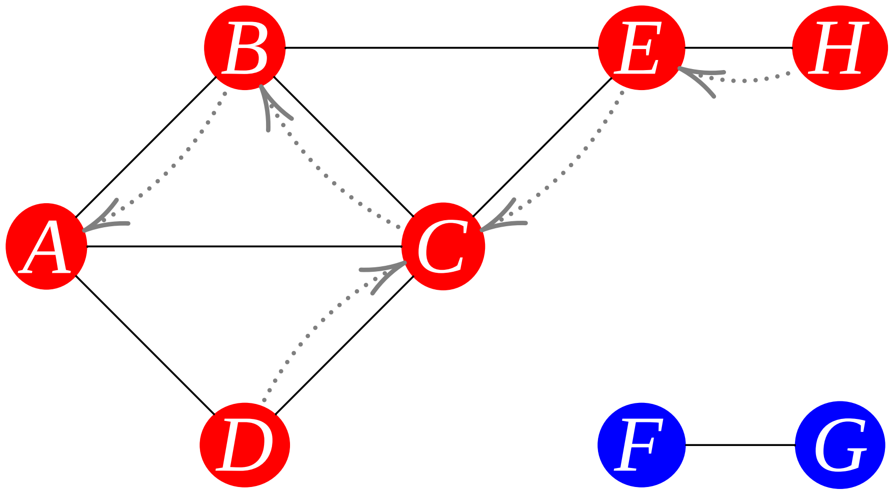
A	B	C	E	H

Depth first search



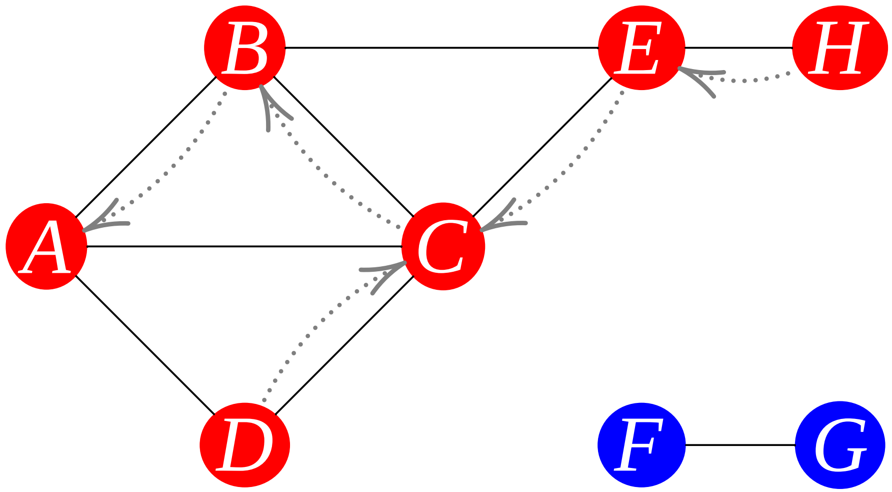
A	B	C	E

Depth first search



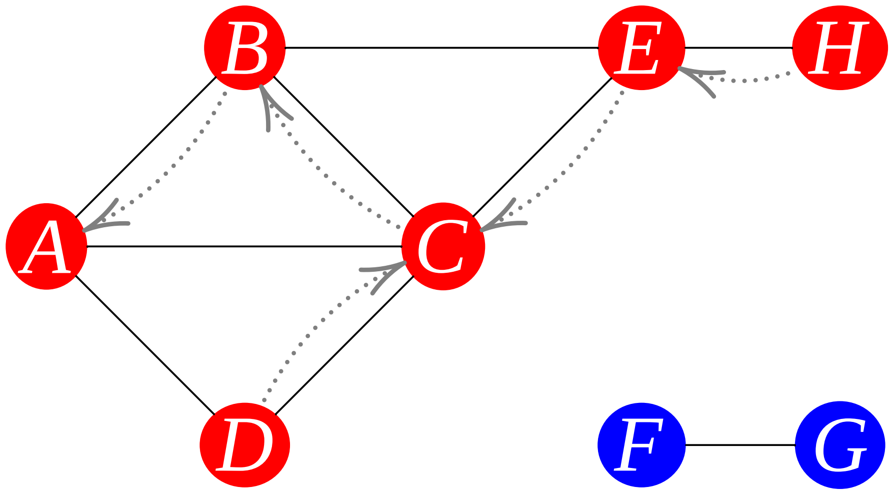
A	B	C

Depth first search



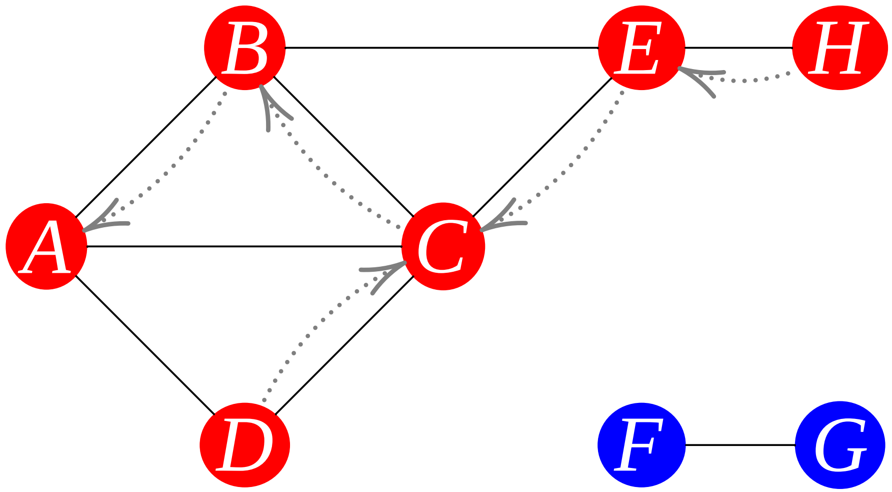
A	B

Depth first search

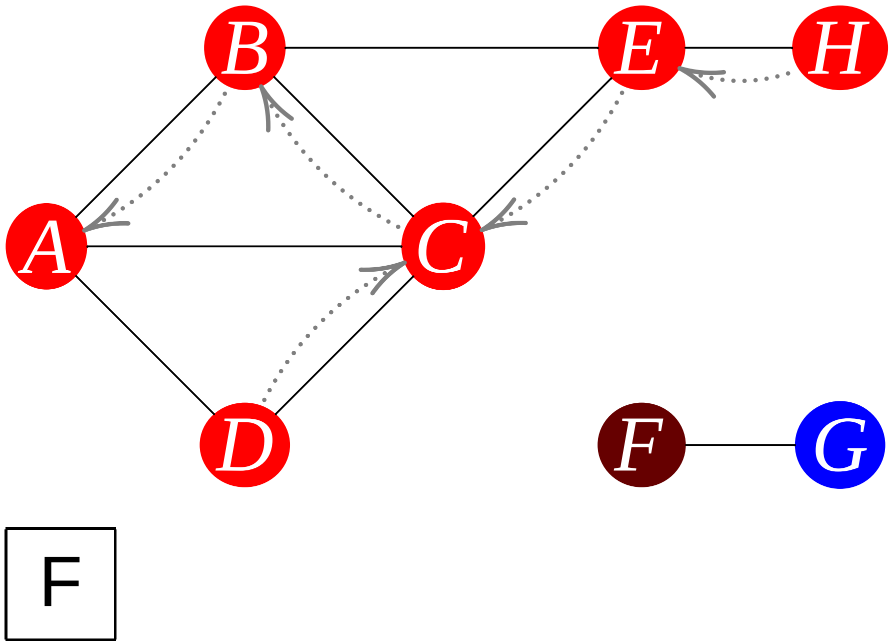


A

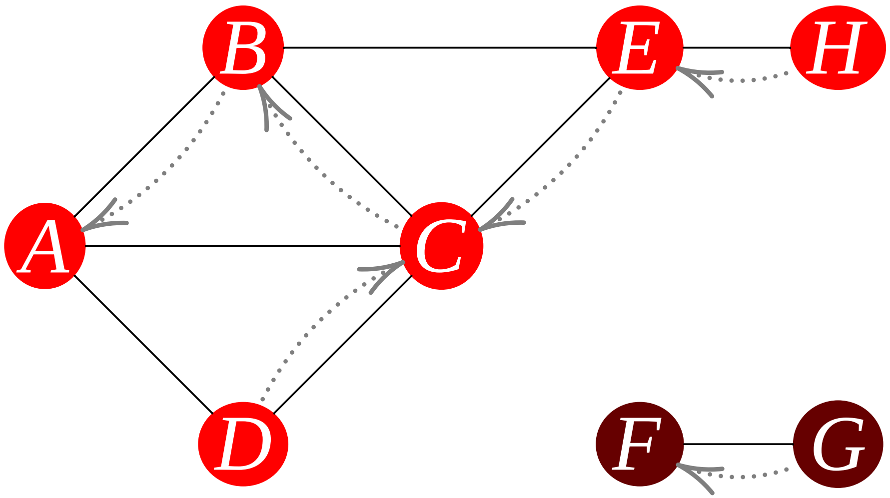
Depth first search



Depth first search

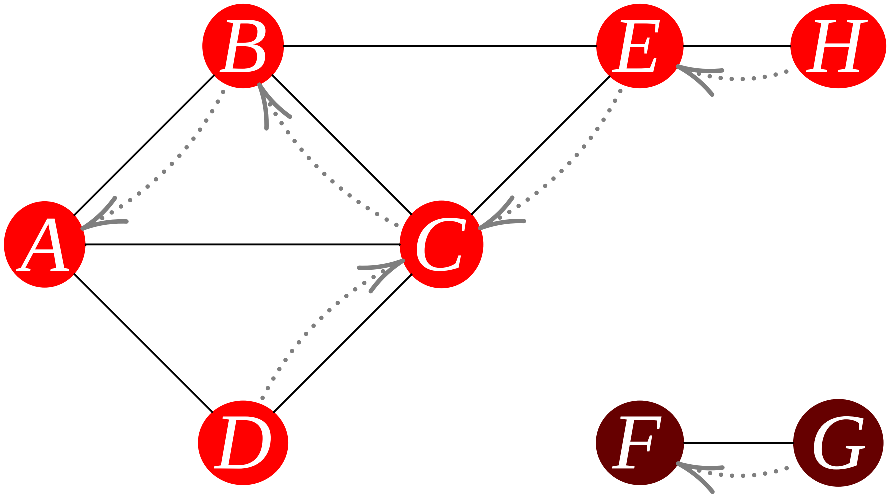


Depth first search



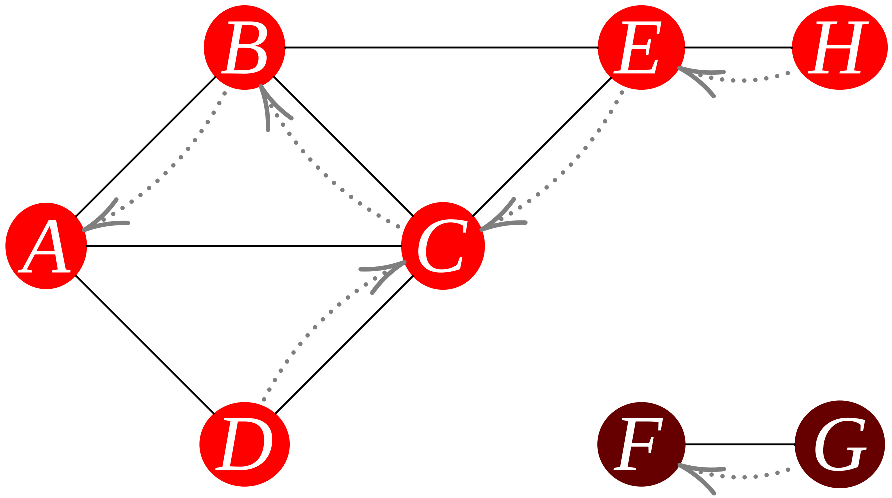
F	G

Depth first search



F

Depth first search



State spaces: examples

- Two robots in a maze, with a single command stream
- A cow in a maze with dynamite to get through walls
- A cow who can only store 5 units of energy, and must replenish at grassy patches
- A ship that takes time to change direction
- A pogo stick that can only gradually change speed

IOI problems

	1	2	3	4	5	6
1999	flower **DP**	codes **string**	under **heur/graph**	lights **shortest**	flatten **maths**	land **DP**
2000	palin **string**	car **maths**	median **sorting**	post **DP**	walls **shortest**	blocks **heur**
2001	mobiles **memory**	ioiwari **minimax**	twofive **maths**	score **minimax**	double **misc**	depot **search**
2002	frog **DP**	utopia **sorting**	xor **heur**	batch **DP**	bus **graph/sort**	rods **misc**
2003	maintain **MST**	code **DP**	reverse **heur**	guess **DP/search**	robots **shortest**	boundary **geom**
2004	hermes **DP**	artemis **DP**	polygon **geometry**	phidias **DP**	farmer **DP**	empodia **misc**

Questions

?